Name: ______________________ Block: __________ Teacher: ______________

Algebra 1

Unit 5 Notes: Comparing Linear, Quadratic, and Exponential Functions

DISCLAIMER: We will be using this note packet for Unit 5. You will be responsible for bringing this packet to class EVERYDAY. If you lose it, you will have to print another one yourself. An electronic copy of this packet can be found on my class blog.

Standards
MGSE9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. • **MGSE9-12.F.LE.1a** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. (This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals). • **MGSE9-12.F.LE.1b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. • **MGSE9-12.F.LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MGSE9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MGSE9-12.F.LE.5 Interpret the parameters in a linear (f(x) = mx + b) and exponential (f(x)=a•dx) function in terms of context. (In the functions above, “m” and “b” are the parameters of the linear function, and “a” and “d” are the parameters of the exponential function.) In context, students should describe what these parameters mean in terms of change and starting value.
MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).
MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.
MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.

Unit 5: Comparing Linear, Quadratic, & Exponential Functions

After completion of this unit, you will be able to…

Learning Target: Comparing Functions in Multiple Representations

- Compare and contrast characteristics of linear, quadratic, and exponential models
- Recognize that exponential and quadratic functions have variable rates of changes whereas linear functions have constant rates of change
- Observe that graphs and tables of exponential functions eventually exceed linear and quadratic functions
- Find and interpret domain and range of linear, quadratic, and exponential functions
- Interpret parameters of linear, quadratic, and exponential functions
- Calculate and interpret average rate of change over a given interval
- Write a function that describes a linear, quadratic, or exponential relationship
- Solve problems in different representations using linear, quadratic, and exponential models
- Construct and interpret arithmetic and geometric sequences

Table of Contents

Timeline for Unit 5

Monday	Tuesday	Wednesday	Thursday	Friday
April 13th	**April 14th** Day 1: Distinguishing between Linear, Quadratic, and Exponential Functions	**15th** Day 2: Characteristics of Functions	**16th** Day 3: Comparing Multiple Representations of Functions	**17th** Day 4: Transformations of Functions

**Unit 5 Test will be given after the EOC

Day 1 – Distinguishing Between Linear, Quadratic, & Exponential Functions

Standard(s):
MGSE9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MGSE9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

In this unit, we will review and compare Linear, Quadratic, and Exponential Functions.

Identifying Types of Functions from an Equation

Classify each equation as linear, quadratic, or exponential:

a. $f(x) = 3x + 2$ b. $y = 5^x$ c. $f(x) = 2$ d. $f(x) = 4(2)^x + 1$ e. $y = 4x^2 + 2x - 1$

Identifying Types of Functions from a Table

- Linear Functions have **constant** (same) **first differences** (add/subtract same number over and over).
- Quadratic Functions have **constant second differences**.
- Exponential functions have **constant ratios** (multiply by same number over and over).

Linear Function

	x	y	
	2	4	
+3	5	3	−1
+3	8	2	−1
+3	11	1	−1

Quadratic Function

x	y		
0	3		
1	2	−1	
2	3	+1	+2
3	6	+3	+2
4	11	+5	+2

Exponential Function

	x	$f(x) = 2(3)^x$	
	1	6	
+1	2	18	×3
+1	3	54	×3
+1	4	162	×3

Determine if the following tables represent linear, quadratic, exponential, or neither and explain why.

a.

x	y
−2	7
−1	4
0	1
1	−2
2	−5

b.

x	y
−1	1.5
0	3
1	6
2	12

c.

x	y
−2	6
−1	3
0	2
1	3
2	6

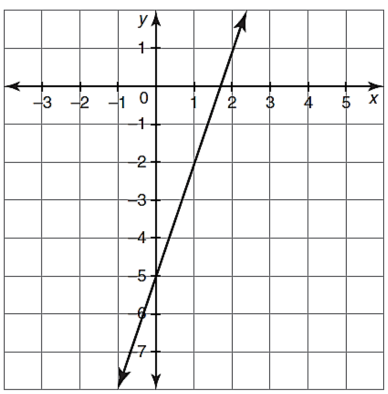
Writing Equations from a Graph or Table

Linear Functions	**Quadratic Functions**	**Exponential Functions**
y = mx + b y = (*slope*)x + *y-intercept* slope = # you add/sub each time y-intercept: starting amount or y-value when x = 0	**$y = a(x-h)^2 + k$** y = *opens*(x – *x-value*)^2 + *y-value* *(h, k) is vertex* **y = a(x – p)(x – q)** y = *opens*(x – *zero*)(x – *zero*) *You then have to multiply your equation out to get to standard form.*	**$y = ab^x$** y = *y-intercept*(*constant ratio*)^x y-intercept: starting amount or y-value when x = 0 constant ratio = # you multiply by each time

For each table or graph below, identify if it is linear, quadratic, or exponential. Then write an equation that represents it.

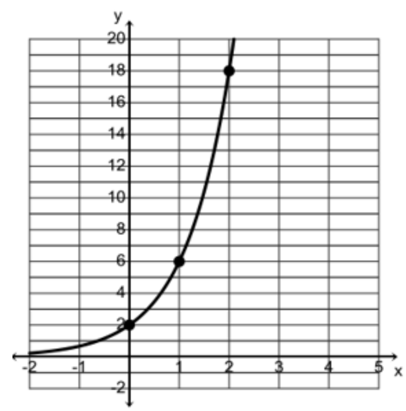
a. Type: ____________________

Equation: __________________



b. Type: ____________________

Equation: __________________



c. Type: ____________________

Equation: __________________

x	–3	–2	–1	0	1	2	3
y	0	5	8	9	8	5	0

d. Type: ____________________

Equation: __________________

x	–3	–2	–1	0	1	2	3
y	–16	–13	–10	–7	–4	–1	2

e. Type: ____________________

Equation: __________________

x	–3	–2	–1	0	1	2	3
y	3	0	–1	0	3	8	15

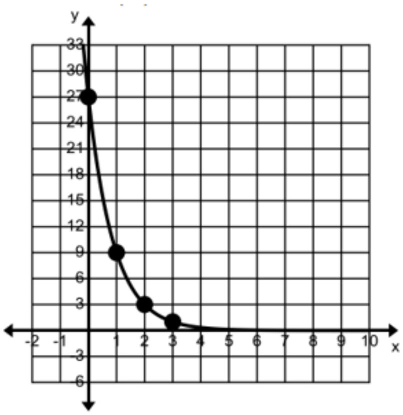
f. Type: ____________________

Equation: __________________

x	0	1	2	3	4	5
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16	64

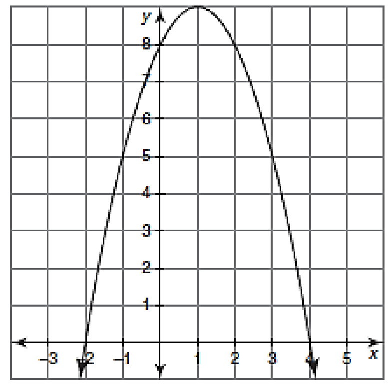
g. Type: ____________________

Equation: __________________



h. Type: ____________________

Equation: __________________



i. Type: ____________________

Equation: __________________

x	–3	–2	–1	0	1	2	3
y	–14	–9	–4	1	6	11	16

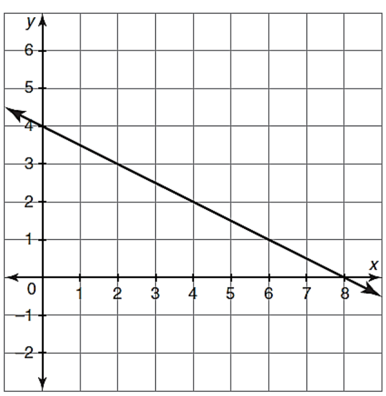
j. Type: ____________________

Equation: __________________

x	–3	–2	–1	0	1	2	3
y	4	8	16	32	64	128	256

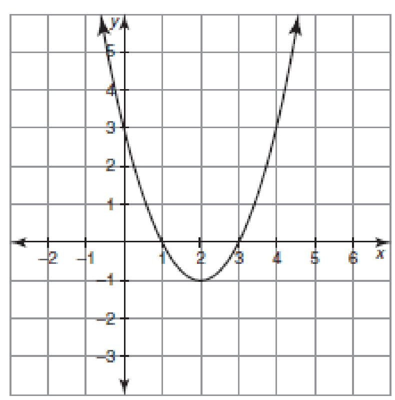
k. Type: ____________________

Equation: __________________



l. Type: ____________________

Equation: __________________

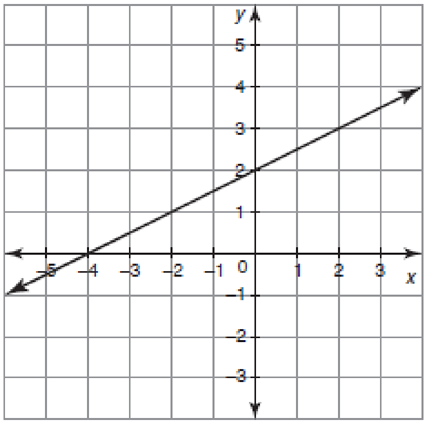


Day 2 – Characteristics of Functions

Standard(s):
MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Which of these characteristics do you already know?

Characteristic	Definition	Notation
Y-Intercept	Where the graph crosses the ____- axis (x = ____)	(0, y)
X-Intercept/ Root/ Zero/ Solution	Where the graph crosses the ____ - axis (y = ____)	(x, 0)
Domain	All the possible ____-values or inputs of a function	All real numbers, ______ (-∞, ∞) or -∞≤ x ≤∞
Range	All the possible ____-values or outputs of a function	y ≤ # or y ≥ #
Vertex	Middle point of the parabola	(x, y)
Axis of Symmetry	_________ that divides the graph into two mirror-images	x = # (x-coordinate of vertex)
Extrema: Maximum/Minimum	Min: ______________ point of a graph Max: _____________ point of a graph	Only for Quadratic Functions
Maximum/Minimum Value	____-value of the maximum or minimum (vertex)	y = # (y-coordinate of vertex)
Intervals of Increase/ Decrease/Constant	Increase: Graph goes_________________ Decrease: Graph goes________________ Constant: Graph _________________	x > # or x < #
Positive/Negative Intervals	Positive: ______________ the x-axis Negative: _____________ the x-axis	# < x < #or x > # or x < #
End Behavior	Where the graph "goes" on the left and right	As x increases... and as x decreases...
Rate of Change	Change in y over change in x Rise over run	$\frac{y_2 - y_1}{x_2 - x_1}$



Domain: ________________ Range: ________________

X-intercept: ________________ Y-intercept: ________________

Zero: ________________ Interval of Constant: __________

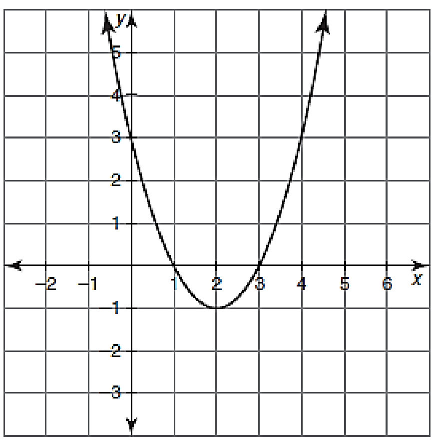
Interval of Increase: __________ Interval of Decrease: __________

Maximum(s): ________________ Minimum(s): ________________

Positive: ________________ Negative: ________________

End Behavior: as $x \to -\infty$, $f(x) \to$ ______
as $x \to \infty$, $f(x) \to$ ______

Rate of Change:



Domain: ________________ Range: ________________

X-intercept: ________________ Y-intercept: ________________

Zero: ________________ Interval of Constant: __________

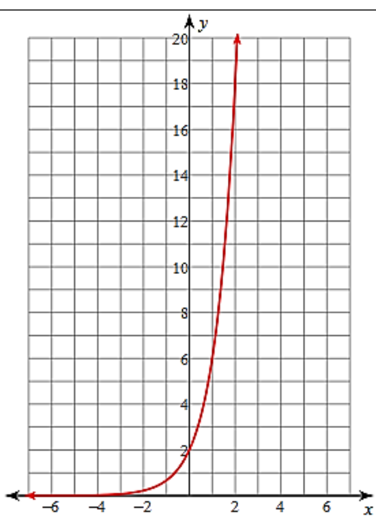
Interval of Increase: __________ Interval of Decrease: __________

Maximum(s): ________________ Minimum(s): ________________

Positive: ________________ Negative: ________________

End Behavior: as $x \to -\infty$, $f(x) \to$ ______
as $x \to \infty$, $f(x) \to$ ______

Rate of Change from $1 \le x \le 4$:



Domain: ________________ Range: ________________

X-intercept: ________________ Y-intercept: ________________

Interval of Increase: __________ Interval of Decrease: __________

Maximum(s): ________________ Minimum(s): ________________

Positive: ________________ Negative: ________________

Asymptote: ________________

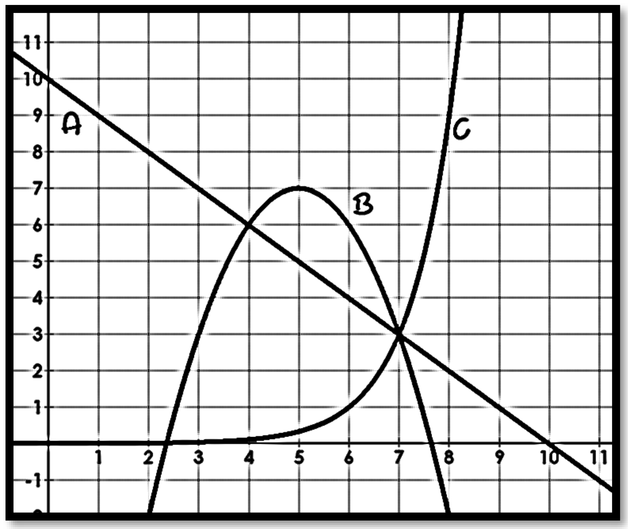
End Behavior: as $x \to -\infty$, $f(x) \to$ ______
as $x \to \infty$, $f(x) \to$ ______

Rate of Change [0, 1]:

Day 3 – Comparing Multiple Representations of Functions

Standard(s):
MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.

Scenario 1: Use the graph below to answer the following questions:



a. Which function has the largest x-intercept?

b. Which function has the largest y-intercept?

c. List the functions in order from smallest to biggest when x = 2:

d. List the functions in order from smallest to biggest when x = 5:

e. List the functions in order from smallest to biggest when x = 7:

f. List the functions in order from smallest to biggest when x = 9:

g. List the functions in order from smallest to biggest when x = 15:

h. Which functions have a positive rate of change throughout the entire graph?

i. Which functions have a negative rate of change throughout the entire graph?

j. Which graph has a rate of change that is negative and positive?

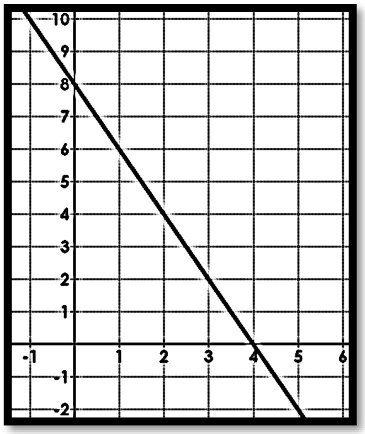
k. Which function has the largest ROC from [3, 5]?

l. Which function has the largest ROC from [7, 8]?

m. Which function will eventually **exceed** the others?

Scenario 2: Consider the following:

f(x)



g(x)

x	$g(x)$
–2	–10
–1	–8
0	–6
1	–4

a. Write an equation for each representation.

b. Which function has the greater y-intercept?

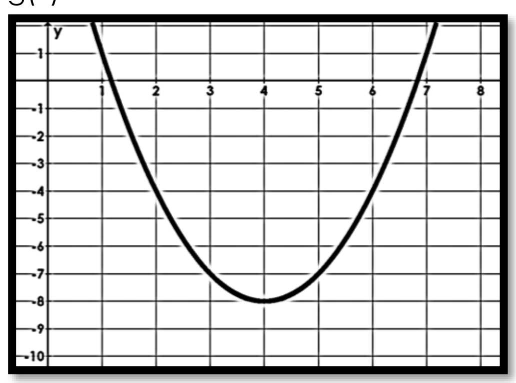
c. Which function has the smaller rate of change?

Scenario 3: Consider the following representations:

a. f(x)

X	-4	-3	-2	-1	0	1
y	0	-5	-8	-9	-8	-5

b. g(x)



a. Which quadratic function has the smaller minimum value? Explain why.

b. Which quadratic function has the bigger y-intercept? Explain why.

c. Name the x-intercepts for each function (estimate if necessary):

f(x): g(x):

4. A table of values is shown for *f*(*x*) and *g*(*x*).

x	$f(x)$
0	0
1	1
2	4
3	9
4	16
5	25

x	$g(x)$
0	–2
1	–1
2	1
3	5
4	13
5	29

Which statement compares the graphs of *f*(*x*) and *g*(*x*) over the interval [0, 5]?

A. The graph of $f(x)$ always exceeds the graph of $g(x)$ over the interval [0, 5].
B. The graph of $g(x)$ always exceeds the graph of $f(x)$ over the interval [0, 5].
C. The graph of $g(x)$ exceeds the graph of $f(x)$ over the interval [0, 4], the graphs intersect at a point between 4 and 5, and then the graph of $f(x)$ exceeds the graph of $g(x)$.
D. The graph of $f(x)$ exceeds the graph of $g(x)$ over the interval [0, 4], the graphs intersect at a point between 4 and 5, and then the graph of $g(x)$ exceeds the graph of $f(x)$.

5. Which statement is true about the graphs of exponential functions?

A. The graphs of exponential functions never exceed the graphs of linear and quadratic functions.
B. The graphs of exponential functions always exceed the graphs of linear and quadratic functions.
C. The graphs of exponential functions eventually exceed the graphs of linear and quadratic functions.
D. The graphs of exponential functions eventually exceed the graphs of linear functions but not quadratic functions.

6. Which statement BEST describes the comparison of the function values for *f*(*x*) and *g*(*x*)?

x	$f(x)$	$g(x)$
0	0	–10
1	2	–9
2	4	–6
3	6	–1
4	8	6

A. The values of $f(x)$ will always exceed the values of $g(x)$.
B. The values of $g(x)$ will always exceed the values of $f(x)$.
C. The values of $f(x)$ exceed the values of $g(x)$ over the interval [0, 5].
D. The values of $g(x)$ begin to exceed the values of $f(x)$ within the interval [4, 5].

Day 4 – Function Transformations

Standard(s): MGSE9-12.F.BF.3
Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. (Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)

Function Notation	**Transformation**	**Function Rule (from $f(x)=x^2$)**
f(x + k)	shift/translation left k units	$f(x) = (x + k)^2$
f(x - k)	shift/translation right k units	$f(x) = (x - k)^2$
f(x) + k	shift/translation up k units	$f(x) = x^2 + k$
f(x) - k	shift/translation down k units	$f(x) = x^2 - k$
kf(x) where k > 1	vertical stretch	$f(x) = kx^2$
kf(x) where k < 1	vertical shrink/compression	$f(x) = kx^2$
-f(x)	reflection over x-axis	$f(x) = -x^2$

1. Suppose the generic function f(x) is transformed such that g(x) = f(x – 2). What transformation best describes the transformation of f(x) to generate g(x)?

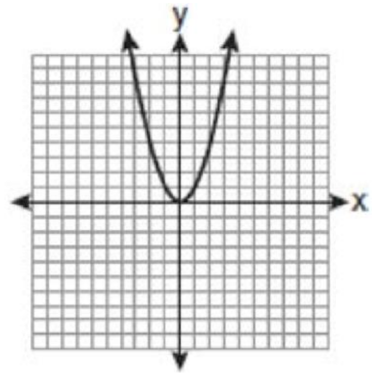
2. Consider the parent function $f(x) = x^2$. The graph of the function $g(x) = -(x + 3)^2 + 5$ is the same as the function f(x) after what transformations?

3. Consider the parent function $f(x) = x^2$. What would be the function rule for g(x) if the graph of g(x) is the same as f(x) after being transformed in the following ways: vertically stretched by a factor of 2, translated left 5 units and down 6 units?

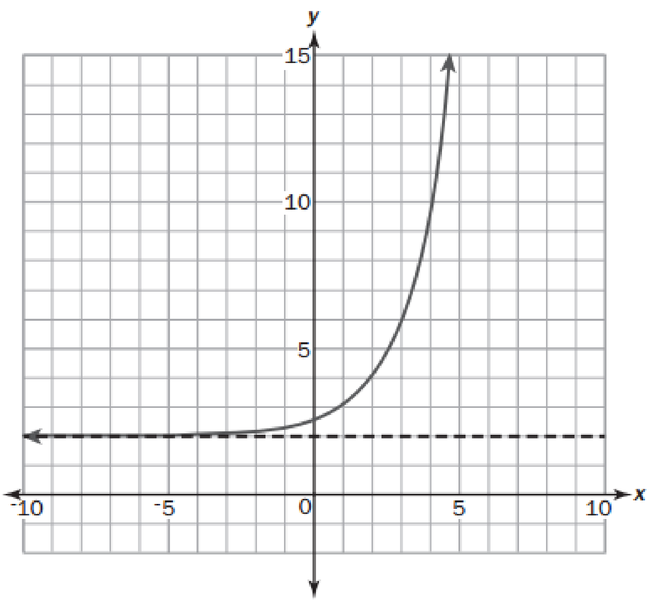
4. Think about the function $f(x) = (x+4)^2 – 7$. What would be the function rule for g(x) if it is translated left 2 units and down 3 units?

5. Think about what the asymptote would be for the function $f(x) = 2^x$. What would be the asymptote of the function $g(x) = 2^x - 4$

6. The graph to the left shows the function f(x). Graph -f(x-5).



7. **Look at the graph.**



Which equation represents this graph?

A. $y = 2^{(x+1)} - 2$
B. $y = 2^{(x-1)} + 2$
C. $y = 2^{(x+2)} - 1$
D. $y = 2^{(x-2)} + 1$

8. **Which function shows the function $f(x) = 3^x$ being translated 5 units to the left?**

A. $f(x) = 3^x - 5$
B. $f(x) = 3^{(x+5)}$
C. $f(x) = 3^{(x-5)}$
D. $f(x) = 3^x + 5$

9. **Which function shows the function $f(x) = 3^x$ being translated 5 units down?**

A. $f(x) = 3^x - 5$
B. $f(x) = 3^{(x+5)}$
C. $f(x) = 3^{(x-5)}$
D. $f(x) = 3^x + 5$

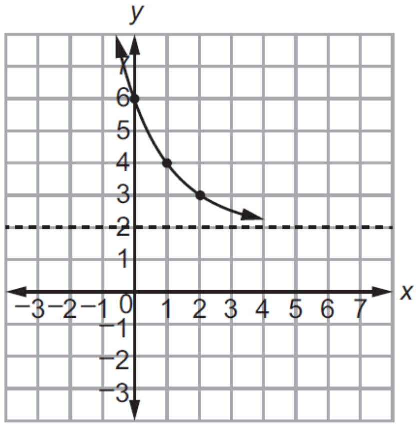
10. **Which statement BEST describes the graph of $f(x + 6)$?**

A. The graph of $f(x)$ is shifted up 6 units.
B. The graph of $f(x)$ is shifted left 6 units.
C. The graph of $f(x)$ is shifted right 6 units.
D. The graph of $f(x)$ is shifted down 6 units.

11. **Which statement BEST describes how the graph of $g(x) = -3x^2$ compares to the graph of $f(x) = x^2$?**

A. The graph of $g(x)$ is a vertical stretch of $f(x)$ by a factor of 3.
B. The graph of $g(x)$ is a reflection of $f(x)$ across the x-axis.
C. The graph of $g(x)$ is a vertical shrink of $f(x)$ by a factor of $\frac{1}{3}$ and a reflection across the x-axis.
D. The graph of $g(x)$ is a vertical stretch of $f(x)$ by a factor of 3 and a reflection across the x-axis.

12. **The graph of the exponential function $f(x) = 4(0.5)^x + 2$ is shown.**



Part A

Which function has the same end behavior as $f(x)$ for large, positive values of x?

A. $g(x) = 4(1.1)^x + 3$
B. $g(x) = 0.5(1.1)^x + 2$
C. $g(x) = 4(0.8)^x + 3$
D. $g(x) = 0.5(0.8)^x + 2$

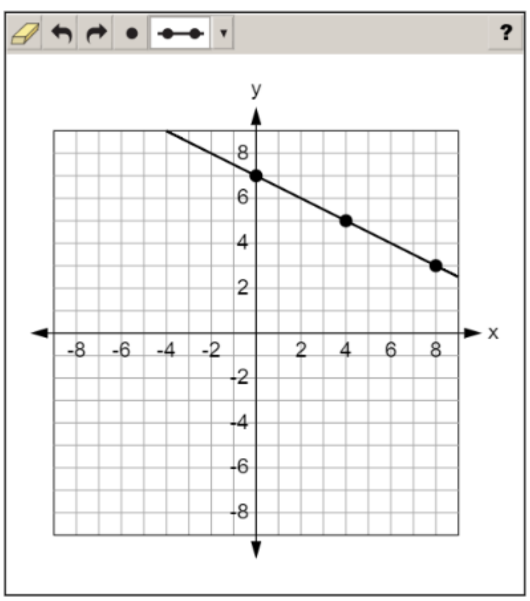
Part B

Which function's graph has a y-intercept of 1?

A. $h(x) = 5(2)^x$
B. $h(x) = 5(0.5)^x + 0.5$
C. $h(x) = (0.5)^x + 1$
D. $h(x) = 0.5(2)^x + 0.5$

13. (Milestones Application Sample Question)

Part A: The graph of f(x) is shown on the coordinate grid. Graph the linear function f(x) – 2



Part B: A linear function g(x) is shown.

$$g(x) = \frac{3}{4}x - 5$$

Graph g(x) + 3